Predictions of Aeroacoustic Characteristics of Coaxial Jets

C. Y. Chen*
United Technologies Research Center, East Hartford, Conn.

Theme

A SIMPLE analytic model is presented for predicting sound power spectral density of coaxial jets at ambient temperature. It is based on a variant of notions of Ribner and Powell together with Reichardt's theory of turbulent mixing. The predictions are in reasonable agreement with experimental data. For a given core jet velocity, the total noise output from a coaxial jet is found to reach a minimum when the outer annular stream velocity is approximately half of the core jet velocity. For a given thrust, the coaxial jet is found to produce minimum noise when the two stream velocities are equal.

Contents

From Lighthill's theory,[1] it can be shown[2-4] that the sound power output per unit length of jet is

$$\frac{\mathrm{d}w}{\mathrm{d}x} \sim \int_0^{r_j} \frac{V_e f_l^4 \overline{T^2}}{a_\infty{}^5 \rho_\infty} 2\pi r \mathrm{d}r \tag{1}$$

when the effects of eddy convection are ignored. Here $\overline{T^2}$ is a typical mean-square value of the quadrupole strength, V_e is the eddy volume, and $f_l(x,r)$ is the frequency of turbulent fluctuations, x and r being the axial and radial coordinates. In this expression, ρ_∞ and a_∞ are the density and speed of sound of the ambient air, respectively, and r_j is the radial distance from the centerline to the outer edge of jet. With the approximations that $V_e \sim \ell^3$, $\ell \sim x$, $f_l \sim \partial U/\partial r$ and $\overline{T^2} \sim \rho^2 U^4$ (ℓ, ρ, and U being the longitudinal length scale of turbulence, mean density, and velocity, respectively), Eq. (1) becomes

$$\frac{\mathrm{d}w}{\mathrm{d}x} \sim \frac{x^3}{\rho_\infty a_\infty^5} \int_0^{r_j} \rho^2 U^4 \left(\frac{\partial U}{\partial r}\right)^4 2\pi r \mathrm{d}r \tag{2}$$

For simplicity of the spectral calculation, one may approximate the sharply peaked spectrum of any axial slice by the central single frequency. This approximation leads to the expression for the sound power spectral density $\mathrm{d}w/\mathrm{d}f$,[3]

$$\frac{\mathrm{d}w}{\mathrm{d}f} = \frac{\mathrm{d}w}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}f} \tag{3}$$

The single characteristic frequency $f(x)$ is assumed to have a similar behavior for each axial slice: $f \sim U_m/\ell$, U_m being the maximum velocity in the axial slice. Since $\ell \sim x$, it follows that

$$f(x) = \frac{\alpha}{2\pi}\frac{U_m(x)}{x} \tag{4}$$

where $\alpha = 14$ is an empirical constant. The sound power spectrum of a jet at ambient temperature can now be written from Eqs. (2)-(4) as

$$\frac{\mathrm{d}w}{\mathrm{d}f} = \beta \frac{\rho_\infty x^4}{a_\infty{}^5 \lambda f} \int_0^{r_j} U^4 \left(\frac{\partial U}{\partial r}\right)^4 2\pi r \mathrm{d}r \tag{5}$$

where β is a proportional constant and

$$\lambda = 1 - \frac{x}{U_m}\frac{\mathrm{d}U_m}{\mathrm{d}x}$$

The mean velocity and its gradient in the jet flowfield are calculated based on Reichardt's theory for free turbulence mixing.[5,6] For a coaxial jet consisting of a core jet of diameter D_1 with velocity U_1 and an outer annular jet of diameter D_2 with velocity U_2, the momentum flux is

$$\overline{\rho u^2}(x,r) = \sum_{i=1}^{2} \frac{\overline{(\rho u^2)}_i}{\pi c_i^2 x^2} \int_{s_i}^{D_i/2} \int_0^{2\pi} \exp\left\{-\frac{r^2+s^2-2rs\cos\gamma}{c_i{}^2 x^2}\right\} s\mathrm{d}s\mathrm{d}\gamma \tag{6}$$

where $s_1 = 0$, $s_2 = D_1/2$, $c_i = 0.0794/(1+0.72\,M_i)$, and M_i is the jet exit Mach number. For a jet at ambient temperature, the density $\rho = \rho_\infty$ can be factored out. If the mean square value $\overline{u^2}$ of the instantaneous velocity is approximated by U^2, the quantity $U(\partial U/\partial r)$ that is required in the acoustic model, Eq. (5) can be obtained by differentiation under the integral signs of Eq. (6).

$$U\frac{\partial U}{\partial r} = \sum_{i=1}^{2} \frac{U_i^2}{\pi c_i^4 x^4} \int_{s_i}^{D_i/2} \int_0^{2\pi} (s\cos\gamma - r) \exp\left\{-\frac{r^2+s^2-2rs\cos\gamma}{c_i^2 x^2}\right\} s\mathrm{d}s\mathrm{d}\gamma \tag{7}$$

Typical results of the spectral calculation are given in Fig. 1. The predicted power spectra for a coaxial jet with area ratio $A_2/A_1 = 5.4$ (A_1 and A_2 being the jet exit areas of the core and outer streams, respectively) and velocity ratios $U_2/U_1 = 0$, 0.5, and 1.0 are in reasonable agreement with the measurements of Olsen.[7] The comparison shows that the model correctly predicts: a) the magnitude and shape of the spectra, b) the shift of spectral peaks as a function of the velocity ratio of the two streams, and c) the crossover of the two spectra at velocity ratios of 0 and 0.5.

An interesting feature of coaxial jet noise can also be described by the present model. At a constant core jet velocity, the measured total sound power output of a coaxial jet[8] decreases as the secondary stream velocity increases from 0 to approximately half of the core jet velocity. The sound power output reaches a minimum at $U_2/U_1 \sim 0.5$ and increases with velocity ratio for $U_2/U_1 > 0.5$. The prediction agrees with this observed trend as shown in Fig. 2, although the noise reduction at velocity ratios less than 0.6 appears to be underestimated.

Presented as Paper 76-4 at the AIAA 14th Aerospace Sciences Meeting, Washington, D. C., Jan. 26-28, 1976; submitted Feb. 9, 1976; synoptic received July 27, 1976; revision received Sept. 29, 1976. Full paper available from AIAA Library, 750 Third Avenue, New York, N.Y. 10017. Price: Microfiche, $2.00; hard copy, $5.00. **Order must be accompanied by remittance.**

Index categories: Aircraft Noise, Power Plant; Jets, Wakes, and Viscid-Inviscid Flow Interactions.

*Senior Research Engineer, Aeroacoustics and Experimental Gas Dynamics Group. Member AIAA.

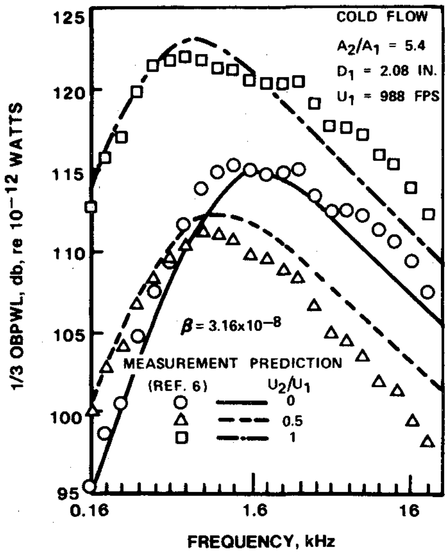

Fig. 1 Sound power spectra of coaxial jet.

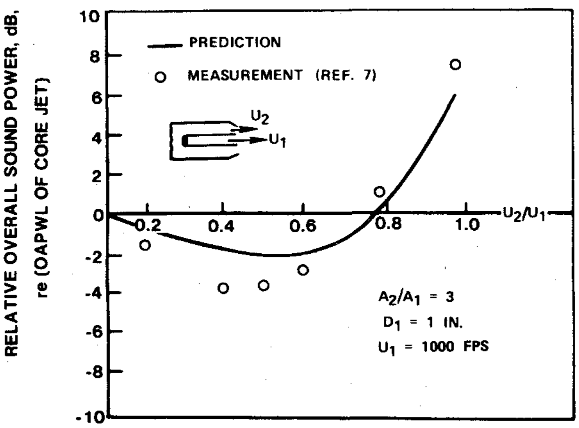

Fig. 2 Relative overall sound power level.

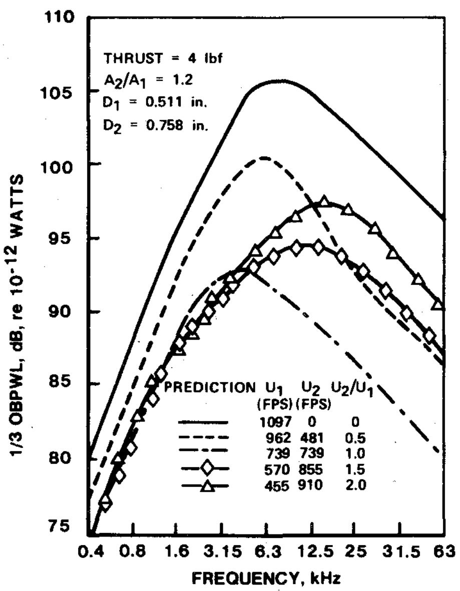

Fig. 3 Predicted sound power spectra at constant thrust.

More relevant to the problem of jet noise suppression is consideration of noise output in terms of thrust. By varying the two stream jet velocities under the constraint of constant

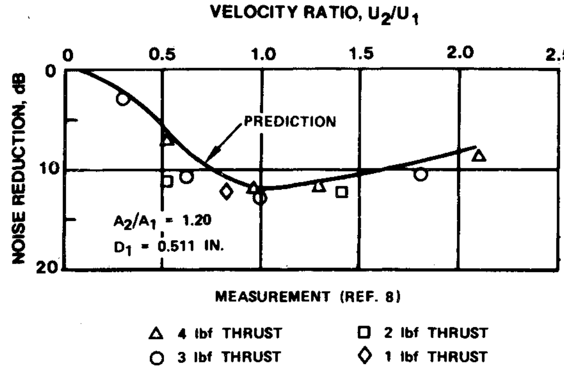

Fig. 4 Jet noise reduction at constant thrust.

thrust, sound power spectra were calculated for a cold jet of area ratio $A_2/A_1 = 1.2$ with $U_2/U_1 = 0, 0.5, 1, 1.5$, and 2 (Fig. 3). At a given thrust of 4 lbf, the highest spectrum shown in the figure corresponds to operating the core jet alone ($U_1 = 1097$ fps, $U_2 = 0$) and the lowest spectrum corresponds to operating a single jet of the larger diameter D_2 ($U_1 = U_2 = 793$ fps). In general, the remaining three spectra fall between. Consequently, at constant thrust, the predicted overall sound power output from a coaxial jet reaches a minimum when the two stream velocities are equal. As shown in Fig. 4, the predicted reduction in overall sound power relative to the core jet alone compares favorably with the measured overall sound pressure reduction at 30° from the jet axis.[9] This comparison is made because measurements of sound power are not available.

Further results are given elsewhere,[10] where it is shown that for coaxial jets the sound source strength distribution along the jet axis (being constant in the initial mixing region and decaying like x^{-7} in the fully turbulent region) and the asymptotic behavior of the spectrum (approaching f^2 and f^{-2} laws in the low-frequency and high-frequency limits, respectively) are identical with the theoretical results for single jets given by several investigators.[2,3] In the spectral calculation, the present model is not limited to the asymptotes and predicts the complete spectra for both single and coaxial jets.

Acknowledgment

The study reported herein was funded by the Pratt & Whitney Aircraft Division.

References

[1] Lighthill, M. J., "On Sound Generated Aerodynamically, I General Theory," *Proceedings of the Royal Society*, Vol. A211, March 1952, pp. 564-587.

[2] Ribner, H. S., "On the Strength Distribution of Noise Sources Along a Jet," UTIA Rept. 51, 1958, Inst. of Aerophysics, Univ. of Toronto, Toronto, Canada.

[3] Powell, A., "Similarity and Turbulent Jet Noise," *Journal of the Acoustical Society of America*, Vol. 31, June 1959, pp. 812-813.

[4] Ribner, H. S., "The Generation of Sound by Turbulent Jets," *Advances in Applied Mechanics*, Vol. 8, 1964, pp. 103-182.

[5] Reichardt, H., "On a New Theory of Free Turbulence," *Journal of the Royal Aeronautical Society*, Vol. 47, June 1943, pp. 167-176.

[6] Alexander, L. G., Baron, T., and Comings, E. W., "Transport of Momentum, Mass and Heat in Turbulent Jets," Univ. of Illinois Engr. Experiment Station, Bulletin Series No. 413.

[7] Olsen, W., NASA Lewis Research Center, Cleveland, Ohio, private communication.

[8] Bielak, G. W., "Coaxial Flow Jet Noise," D6E-10041-1, 1972, Boeing/Aeritalia Co., Seattle, Wash.

[9] Williams, T. J., Ali, M. R. M., and Anderson, J. S., "Noise and Flow Characteristics of Coaxial Jets," *Journal of Mechanics and Engineering Science*, Vol. 11, No. 2, April 1969, pp. 133-142.

[10] Chen, C. Y., "A Model for Predicting Aeroacoustic Characteristics of Coaxial Jets," AIAA Paper 76-4, 1976.